KEK-TH-438
KEK-preprint 95-156
November 1995

Formulations of spin 1 resonances in the chiral lagrangian

Masaharu Tanabashi*
National Laboratory for High Energy Physics (KEK)
Tsukuba, Ibaraki 305, Japan

Abstract

Equivalence of the hidden local symmetry formulation with non-minimal interactions and the anti-symmetric tensor field method of ρ and a_1 mesons in the chiral lagrangian is shown by using the auxiliary field method. Violation of the KSRF I relation, which becomes important in the application of chiral lagrangian to *non QCD-like* technicolor models can be parametrized by non-minimal coupling in the hidden local symmetry formalism keeping low energy theorem of hidden local symmetry. We also obtain explicit correspondence of parameters in both formulations.

The vector meson plays important roles in the chiral lagrangian of the spontaneous chiral symmetry breaking in QCD. The observed sizes of chiral coefficients $L_{1,2,3,9,10}$ in one loop chiral perturbation theory[1] at $\mathcal{O}(E^4)$ are saturated by the vector meson contribution[2].

In the application of the chiral lagrangian to the strongly interacting Higgs sector, the chiral coefficients L_{10} and L_9 correspond[3] to the Peskin–Takeuchi S parameter[4] and the anomalous triple gauge boson interactions $\Delta\kappa_{\gamma,Z}$[5]. Thus, the measurements of

*E-mail address: tanabash@theory.kek.jp

these parameters give severe mass bounds on the techni-ρ of the QCD-like technicolor model.

It should be emphasized, however, that the naive QCD-like technicolor models already suffer from the serious disease of excess of flavor changing neutral currents. We thus need to consider *non QCD-like* technicolor models, e.g., the walking technicolor model[6] and the technicolor model with an elementary scalar[7], etc.. Unlike the naive QCD-like technicolor model, these non QCD-like models are considered to have relatively hard high energy behavior of the Nambu-Goldstone boson form factor due to the large anomalous dimension or the appearance of the elementary scalar.

Many formulations to incorporate the ρ meson into the chiral lagrangian have been proposed. One of the most famous formulations was proposed by Bando, Kugo, Uehara, Yamawaki and Yanagida (BKUYY), in which the ρ meson is treated as a gauge field of "hidden local symmetry" in the chiral lagrangian[8, 9]. In the absence of the external gauge fields (W and photon), the BKUYY formulation has two free parameters (a, g) in addition to f_π. It thus describes the most general amplitude of the $\rho\pi\pi$ coupling and the mass of the ρ-meson. The BKUYY formulation, however, fixes the amplitude of the mixing of the external gauge field and the ρ meson field, leading to the KSRF[10] I relation[11, 12, 13].

The "vector limit" model of the ρ meson[14] can be considered as a special case of this model ($a = 1$). An one loop calculation is performed based on the BKUYY formulation[13, 15]. The technicolored version of this model is known as the BESS model[16]. Despite the great success of this model in QCD and QCD-like technicolor, the BKUYY formulation is not appropriate for the analysis of the non QCD-like technicolor model as it stands, since the KSRF I relation in QCD is a manifestation of the soft high energy behavior of the pion form factor.

Yet another popular formulation of the ρ meson was proposed by Gasser and Leutwyler[1, 2], in which the ρ meson is represented by an anti-symmetric tensor field. In the absence of external gauge fields, this formulation has two parameters (G_V, M_V) corresponding to the $\rho\pi\pi$ coupling and the mass of the ρ meson in addition to f_π. This model is equivalent to the usual vector field formulation including hidden local symmetry formulation in the absence of the external gauge field in a Hamiltonian language[18].

Unlike the hidden local symmetry formalism, the ρ-photon mixing amplitude is left to be a free parameter F_V in this model. Although this extended parameter space is suited for the analysis of non QCD-like technicolor model, actual calculations e.g. one

loop chiral logarithms, are difficult in this model due to its complicated Feynman rules.

It has been checked that the vector meson contribution to the low energy chiral coefficients $L_{1,2,3,9,10}$ are independent of the choice of the formulations for the case of QCD[17] where the KSRF I relation is known to be satisfied phenomenologically. However, the difference of the formulations becomes serious in the *non QCD-like* technicolor models. Actually the BKUYY formulation predicts $L_{10} = -L_9$ as a consequence of the KSRF I relation, which leads to serious cancellations in $\Delta\kappa_{\gamma,Z}$[3], while the anti-symmetric tensor method does not give such a prediction.

The aim of this paper is to give a simple method to study the relation between both formulations by using an auxiliary field method. We find that the anti-symmetric tensor method becomes equivalent to the hidden local symmetry formalism after adding several $\mathcal{O}(E^4)$ parameters in the hidden local lagrangian.

For simplicity, we first study the effective lagrangian of the π and the ρ meson without including the a_1 meson. The effect of the a_1 meson will be discussed later.

Let us start with the conventional chiral lagrangian of $SU(N)_L \times SU(N)_R/SU(N)_V$ symmetry so as to fix our notations:

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}((D^\mu U)^\dagger (D_\mu U)), \qquad U = \exp(2i\frac{\pi^a T^a}{f_\pi}), \tag{1}$$

with T^a the $SU(N)$ generator. Here the chiral field U transforms as $U \to g_L U g_R^\dagger$ under $SU(N)_L \times SU(N)_R$. The covariant derivative D_μ is given by

$$D_\mu U = \partial_\mu U - i\mathcal{L}_\mu U + iU\mathcal{R}_\mu.$$

Here, we introduced the external gauge fields $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ corresponding to the chiral symmetry $SU(N)_L \times SU(N)_R$.

In ref[1, 2] a ρ meson field is introduced as an anti-symmetric tensor field $\mathbf{V}_{\mu\nu}$ with the matter–type transformation properties:

$$\begin{aligned}\mathcal{L}_{\mathrm{AST}} \;=\; & \frac{f_\pi^2}{4}\mathrm{tr}((D^\mu U)^\dagger (D_\mu U)) - \frac{1}{2}\mathrm{tr}(\nabla^\lambda \mathbf{V}_{\lambda\mu}\nabla_\nu \mathbf{V}^{\nu\mu}) + \frac{M_V^2}{4}\mathrm{tr}(\mathbf{V}_{\mu\nu}\mathbf{V}^{\mu\nu}) \\ & + \frac{F_V}{\sqrt{2}}\mathrm{tr}(\mathbf{V}^{\mu\nu}\hat{\mathcal{V}}_{\mu\nu}) + \frac{G_V}{2\sqrt{2}} i\mathrm{tr}(\mathbf{V}^{\mu\nu}[u_\mu, u_\nu]).\end{aligned} \tag{2}$$

We define ξ as

$$U = \xi\xi, \tag{3}$$

and the covariant derivative ∇_μ is given by

$$\nabla^\lambda \mathbf{V}_{\lambda\mu} = \partial^\lambda \mathbf{V}_{\lambda\mu} + [\Gamma^\lambda, \mathbf{V}_{\lambda\mu}], \quad \Gamma_\mu = -\frac{1}{2}(\partial_\mu \xi^\dagger \cdot \xi + \partial_\mu \xi \cdot \xi^\dagger + i\xi^\dagger \mathcal{L}_\mu \xi + i\xi \mathcal{R}_\mu \xi^\dagger).$$

The chiral covariant one form u_μ and the external vector field strength $\hat{\mathcal{V}}_{\mu\nu}$ are defined by

$$u_\mu = i\xi^\dagger (D_\mu U)\xi^\dagger, \qquad \hat{\mathcal{V}}_{\mu\nu} = \frac{1}{2}(\xi^\dagger \mathcal{L}_{\mu\nu}\xi + \xi \mathcal{R}_{\mu\nu}\xi^\dagger).$$

This model has four parameters f_π, M_V, F_V, G_V corresponding to the pion decay constant, the mass of the ρ meson, ρ-γ mixing and $\rho\pi\pi$ coupling, respectively. The KSRF I relation is expressed by the relation $F_V = 2G_V$.

The space components of the anti-symmetric tensor field $\mathbf{V}_{ij} = -\mathbf{V}_{ji}$ $(i, j = 1, 2, 3)$ do not have time derivatives and thus they can be removed from the dynamics. The time components $\mathbf{V}_{0i} = -\mathbf{V}_{i0}$ are the dynamical degrees of freedom identified as the ρ meson.

Bando, Kugo, Uehara, Yamawaki and Yanagida (BKUYY)[8] noted that the decomposition (3) to include matter fields in the chiral lagrangian can be extended to the following form:

$$U = \xi_L^\dagger \xi_R. \tag{4}$$

The decomposition (4) has an ambiguity in the definition of ξ_L and ξ_R. BKUYY regarded this ambiguity as a symmetry (hidden local symmetry, $H = SU(N)$)

$$\xi_L \to h\xi_L g_L^\dagger, \qquad \xi_R \to h\xi_R g_R^\dagger, \qquad h \in H. \tag{5}$$

By introducing the ρ meson as a gauge field of the above hidden local symmetry, BKUYY proposed a lagrangian:

$$\mathcal{L}_{\rm BKUYY} = f_\pi^2 {\rm tr}(\hat{\alpha}_{\mu\perp}\hat{\alpha}_\perp^\mu) + a f_\pi^2 {\rm tr}(\hat{\alpha}_{\mu\parallel}\hat{\alpha}_\parallel^\mu) - \frac{1}{2g^2}{\rm tr}(V_{\mu\nu}V^{\mu\nu}), \tag{6}$$

where $\hat{\alpha}_{\mu\perp}$ and $\hat{\alpha}_{\mu\parallel}$ are given by

$$\hat{\alpha}_{\mu\perp} \equiv \frac{1}{2i}\left(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger\right), \qquad \hat{\alpha}_{\mu\parallel} \equiv \frac{1}{2i}\left(D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger\right),$$

with the covariant derivative D_μ being

$$D_\mu \xi_L = \partial_\mu \xi_L - iV_\mu \xi_L + i\xi_L \mathcal{L}_\mu, \qquad D_\mu \xi_R = \partial_\mu \xi_R - iV_\mu \xi_R + i\xi_R \mathcal{R}_\mu.$$

The gauge field V_μ is identified with the ρ meson.

This model is parametrized by 3 parameters (f_π, a, g), while it explains 4 physical quantities (the pion decay constant, the ρ-γ mixing, the mass of the ρ meson and the $\rho\pi\pi$ couplings). This model thus has one prediction, corresponding to the KSRF I relation. Actually the KSRF I at zero momentum is derived as a low energy theorem of the hidden local symmetry. However, it should be noted that the off-shell relation in the effective field theory might be unphysical because it depends on the definition of the effective fields. Actually as we will show later, the physical on-shell KSRF I relation can be violated by adding higher derivative terms keeping manifest hidden local symmetry.

We note here that the hidden local symmetry can be made manifest without introducing a corresponding gauge field. Actually, by changing the definition of Γ_μ, u_μ and $\hat{\mathcal{V}}_{\mu\nu}$:

$$\begin{aligned}
\Gamma_\mu &= -\frac{1}{2}(\partial_\mu \xi_L \cdot \xi_L^\dagger + \partial_\mu \xi_R \cdot \xi_R^\dagger + i\xi_L \mathcal{L}_\mu \xi_L^\dagger + i\xi_R \mathcal{R}_\mu \xi_R^\dagger), \\
u_\mu &= i\xi_L (D_\mu U)\xi_R^\dagger, \\
\hat{\mathcal{V}}_{\mu\nu} &= \frac{1}{2}(\xi_L \mathcal{L}_{\mu\nu}\xi_L^\dagger + \xi_R \mathcal{R}_{\mu\nu}\xi_R^\dagger)
\end{aligned}$$

the anti-symmetric tensor lagrangian (2) becomes invariant under the hidden local symmetry. In this case Γ_μ plays the role of the gauge connection of the hidden local symmetry. We further obtain the following relations between the anti-symmetric tensor and the BKUYY notations:

$$\Gamma_\mu = -i(\hat{\alpha}_{\mu\parallel} + V_\mu), \qquad u_\mu = 2\hat{\alpha}_{\mu\perp}.$$

Now, we are ready to show the equivalence of both formulations. We introduce an auxiliary field V_μ into the lagrangian (2) of the anti-symmetric tensor formalism. The dynamics is not modified by adding an auxiliary field V_μ:

$$\mathcal{L}'_{\rm AST} = \mathcal{L}_{\rm AST} + \frac{\kappa^2}{2}{\rm tr}\left((V_\mu - i\Gamma_\mu - \frac{1}{\kappa}\nabla^\nu \mathbf{V}_{\nu\mu})^2\right), \tag{7}$$

with κ an arbitrary parameter. The lagrangian (7) then reads:

$$\begin{aligned}
\mathcal{L}'_{\rm AST} &= f_\pi^2 {\rm tr}(\hat{\alpha}_{\mu\perp}\hat{\alpha}_\perp^\mu) + \kappa {\rm tr}(\hat{\alpha}_\parallel^\nu \nabla^\mu \mathbf{V}_{\mu\nu}) + \frac{\kappa^2}{2}{\rm tr}(\hat{\alpha}_{\mu\parallel}\hat{\alpha}_\parallel^\mu) + \frac{M_V^2}{4}{\rm tr}(\mathbf{V}_{\mu\nu}\mathbf{V}^{\mu\nu}) \\
&\quad + \frac{F_V}{\sqrt{2}}{\rm tr}(\mathbf{V}_{\mu\nu}\hat{\mathcal{V}}^{\mu\nu}) + \sqrt{2}G_V i{\rm tr}(\mathbf{V}_{\mu\nu}[\hat{\alpha}_\perp^\mu, \hat{\alpha}_\perp^\nu]),
\end{aligned} \tag{8}$$

Performing a partial integral, we can remove the derivative term of the anti-symmetric tensor field:

$${\rm tr}(\hat{\alpha}_\parallel^\nu \nabla^\mu \mathbf{V}_{\mu\nu}) = -\frac{1}{2}{\rm tr}((D^\mu \hat{\alpha}_\parallel^\nu - D^\nu \hat{\alpha}_\parallel^\mu)\mathbf{V}_{\mu\nu}) + i{\rm tr}([\hat{\alpha}_\parallel^\mu, \hat{\alpha}_\parallel^\nu]\mathbf{V}_{\mu\nu}), \tag{9}$$

where we have defined the covariant derivative $D_\mu \hat{\alpha}_{\nu\parallel} \equiv \partial_\mu \hat{\alpha}_{\nu\parallel} - i[V_\mu, \hat{\alpha}_{\nu\parallel}]$. We note here that the partial integral (9) transfers the dynamical degree of freedom from the anti-symmetric tensor field $\mathbf{V}_{\mu\nu}$ to the auxiliary field V_μ. Plugging an identity

$$D_\mu \hat{\alpha}_{\nu\parallel} - D_\nu \hat{\alpha}_{\mu\parallel} = i[\hat{\alpha}_{\mu\parallel}, \hat{\alpha}_{\nu\parallel}] + i[\hat{\alpha}_{\mu\perp}, \hat{\alpha}_{\nu\perp}] + \hat{\mathcal{V}}_{\mu\nu} - V_{\mu\nu},$$

with $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu]$ into (9), we find

$$\mathrm{tr}(\hat{\alpha}^\nu_\parallel \nabla^\mu \mathbf{V}_{\mu\nu}) = \frac{i}{2}\mathrm{tr}([\hat{\alpha}^\mu_\parallel, \hat{\alpha}^\nu_\parallel]\mathbf{V}_{\mu\nu}) - \frac{i}{2}\mathrm{tr}([\hat{\alpha}^\mu_\perp, \hat{\alpha}^\nu_\perp]\mathbf{V}_{\mu\nu}) - \frac{1}{2}\hat{\mathcal{V}}_{\mu\nu} + \frac{1}{2}V_{\mu\nu}.$$

The lagrangian (8) then reads

$$\begin{aligned} \mathcal{L}'_{\mathrm{AST}} &= f_\pi^2 \mathrm{tr}(\hat{\alpha}_{\mu\perp}\hat{\alpha}^\mu_\perp) + \frac{M_V^2}{4}\mathrm{tr}(\mathbf{V}_{\mu\nu}\mathbf{V}^{\mu\nu}) \\ &+ \frac{i}{2}\kappa \mathrm{tr}(\mathbf{V}^{\mu\nu}[\hat{\alpha}_{\mu\parallel}, \hat{\alpha}_{\nu\parallel}]) - i\left(\frac{\kappa}{2} - \sqrt{2}G_V\right)\mathrm{tr}(\mathbf{V}^{\mu\nu}[\hat{\alpha}_{\mu\perp}, \hat{\alpha}_{\nu\perp}]) \\ &+ \frac{\kappa}{2}\mathrm{tr}(\mathbf{V}^{\mu\nu}V_{\mu\nu}) - \left(\frac{\kappa}{2} - \frac{F_V}{\sqrt{2}}\right)\mathrm{tr}(\mathbf{V}^{\mu\nu}\hat{\mathcal{V}}_{\mu\nu}) + \frac{\kappa^2}{2}\mathrm{tr}(\hat{\alpha}_{\mu\parallel}\hat{\alpha}^\mu_\parallel). \end{aligned} \tag{10}$$

It is easy to integrate out the anti-symmetric tensor field $\mathbf{V}_{\mu\nu}$:

$$\begin{aligned} \mathcal{L}'_{\mathrm{AST}} &= f_\pi^2 \mathrm{tr}(\hat{\alpha}_{\mu\perp}\hat{\alpha}^\mu_\perp) + \frac{\kappa^2}{2}\mathrm{tr}(\hat{\alpha}_{\mu\parallel}\hat{\alpha}^\mu_\parallel) \\ &- \frac{1}{4M_V^2}\mathrm{tr}\left(\left(\kappa V_{\mu\nu} - (\kappa - \sqrt{2}F_V)\hat{\mathcal{V}}_{\mu\nu} + \kappa i[\hat{\alpha}_{\mu\parallel}, \hat{\alpha}_{\nu\parallel}] - (\kappa - 2\sqrt{2}G_V) i[\hat{\alpha}_{\mu\perp}, \hat{\alpha}_{\nu\perp}]\right)^2\right). \end{aligned} \tag{11}$$

The lagrangian (11) is equivalent to the BKUYY lagrangian with several extra terms:

$$\begin{aligned} \mathcal{L}_{\mathrm{BKUYY}} &= f_\pi^2 \mathrm{tr}(\hat{\alpha}_{\mu\perp}\hat{\alpha}^\mu_\perp) + a f_\pi^2 \mathrm{tr}(\hat{\alpha}_{\mu\parallel}\hat{\alpha}^\mu_\parallel) - \frac{1}{2g^2}\mathrm{tr}(V_{\mu\nu}V^{\mu\nu}) \\ &+ z_1 \mathrm{tr}(\hat{\mathcal{V}}_{\mu\nu}\hat{\mathcal{V}}^{\mu\nu}) + z_3 \mathrm{tr}(\hat{\mathcal{V}}_{\mu\nu}V^{\mu\nu}) + z_4 i \mathrm{tr}(V_{\mu\nu}\hat{\alpha}^\mu_\perp \hat{\alpha}^\nu_\perp) + z_5 i \mathrm{tr}(V_{\mu\nu}\hat{\alpha}^\mu_\parallel \hat{\alpha}^\nu_\parallel) \\ &+ z_6 i \mathrm{tr}(\hat{\mathcal{V}}_{\mu\nu}\hat{\alpha}^\mu_\perp \hat{\alpha}^\nu_\perp) + z_7 i \mathrm{tr}(\hat{\mathcal{V}}_{\mu\nu}\hat{\alpha}^\mu_\parallel \hat{\alpha}^\nu_\parallel) + \cdots, \end{aligned} \tag{12}$$

with

$$\begin{aligned} a &= \frac{\kappa^2}{2f_\pi^2}, & g &= \frac{\sqrt{2}M_V}{\kappa}, \\ z_1 &= -\frac{(\kappa - \sqrt{2}F_V)^2}{4M_V^2}, & z_3 &= \frac{\kappa(\kappa - \sqrt{2}F_V)}{2M_V^2}, \\ z_4 &= \frac{\kappa(\kappa - 2\sqrt{2}G_V)}{M_V^2}, & z_5 &= -\frac{\kappa^2}{M_V^2}, \\ z_6 &= -\frac{(\kappa - \sqrt{2}F_V)(\kappa - 2\sqrt{2}G_V)}{M_V^2}, & z_7 &= \frac{\kappa(\kappa - \sqrt{2}F_V)}{M_V^2}, \end{aligned}$$

where we have used the notation of Ref[15] and $\cdots$ in (12 stands for operators corresponding to four point vertices.

The above procedure, however, leaves the artificial coefficient κ. What is the meaning of κ, then? Since we are dealing with an effective theory, there is an ambiguity in the definition of effective fields. The redefinition of the effective field does not change the physical on-shell S matrix, even though it modifies parameters in the effective lagrangian. The arbitrary parameter κ corresponds to this superficial parameter difference as we will show in the next paragraph.

Let us consider the redefinition of the ρ meson field in the hidden local symmetry formulation:

$$V_\mu \to V_\mu + (1-K)\hat{\alpha}_{\mu\parallel}.$$

This redefinition leads to:

$$\begin{aligned} V_{\mu\nu} &\to KV_{\mu\nu} + (1-K)\hat{\mathcal{V}}_{\mu\nu} + K(1-K)i[\hat{\alpha}_{\mu\parallel}, \hat{\alpha}_{\nu\parallel}] + (1-K)i[\hat{\alpha}_{\mu\perp}, \hat{\alpha}_{\nu\perp}], \\ \hat{\alpha}_{\mu\parallel} &\to K\hat{\alpha}_{\mu\parallel}. \end{aligned} \tag{13}$$

Plugging (13) into (11), we find that the κ dependence of (11) appears only in the form κK. The arbitrary parameter K thus actually corresponds to the ambiguity of κ.

What is the most convenient choice of the ρ meson field definition, then? One plausible choice is to define the ρ meson field so as to eliminate one of the non-minimal couplings $z_{1,3,4,5,6,7}$. In the following, we choose a ρ meson field definition in which the kinetic ρ-γ mixing z_3 is absent ($\kappa = \sqrt{2}F_V$).

This particular choice of the ρ meson field definition resolves the ambiguity of κ. We find the following relations:

$$\begin{aligned} a &= \frac{F_V^2}{f_\pi^2}, & g &= \frac{M_V}{F_V}, \\ z_1 &= 0, & z_3 &= 0, \\ z_4 &= \frac{2F_V}{M_V^2}(F_V - 2G_V), & z_5 &= -\frac{2F_V^2}{M_V^2}, \\ z_6 &= 0, & z_7 &= 0. \end{aligned} \tag{14}$$

Note here that the violation of the KSRF I relation $F_V = 2G_V$ in the anti-symmetric tensor formalism leads to the appearance of the non-minimal $\rho\pi\pi$ coupling z_4. This coupling actually violates the physical KSRF I relation, while it does not contribute to the $\rho\pi\pi$ coupling at zero momentum keeping the low energy theorem of the hidden local symmetry[12, 13].

We next discuss the axial-vector meson (the a_1 meson) in the chiral lagrangian. In the anti-symmetric tensor field method, it is straightforward to introduce the a_1 meson[2, 17]:

$$\begin{aligned}
\mathcal{L}_{\rm AST} &= \frac{f_\pi^2}{4}{\rm tr}((D^\mu U)^\dagger (D_\mu U)) \\
&-\frac{1}{2}{\rm tr}(\nabla^\lambda \mathbf{A}_{\lambda\mu}\nabla_\nu \mathbf{A}^{\nu\mu}) + \frac{M_A^2}{4}{\rm tr}(\mathbf{A}_{\mu\nu}\mathbf{A}^{\mu\nu}) - \frac{F_A}{\sqrt{2}}{\rm tr}(\mathbf{A}^{\mu\nu}\hat{\mathcal{A}}_{\mu\nu}), \\
&-\frac{1}{2}{\rm tr}(\nabla^\lambda \mathbf{V}_{\lambda\mu}\nabla_\nu \mathbf{V}^{\nu\mu}) + \frac{M_V^2}{4}{\rm tr}(\mathbf{V}_{\mu\nu}\mathbf{V}^{\mu\nu}) + \frac{F_V}{\sqrt{2}}{\rm tr}(\mathbf{V}^{\mu\nu}\hat{\mathcal{V}}_{\mu\nu}) \\
&+\frac{G_V}{2\sqrt{2}} i{\rm tr}(\mathbf{V}^{\mu\nu}[u_\mu, u_\nu])
\end{aligned} \tag{15}$$

where M_A and F_A are the mass and the decay constant of the a_1 meson respectively, and $\hat{\mathcal{A}}_{\mu\nu}$ is defined by

$$\hat{\mathcal{A}}_{\mu\nu} = \frac{1}{2}(-\xi_L \mathcal{L}_{\mu\nu}\xi_L^\dagger + \xi_R \mathcal{R}_{\mu\nu}\xi_R^\dagger).$$

The anti-symmetric tensor field $\mathbf{A}_{\mu\nu}$ represents the a_1 meson.

Bando, Kugo and Yamawaki (BKY) introduced the a_1 meson as a gauge field of generalized hidden local symmetry $(H_L \times H_R = SU(N)\times SU(N))$[11]:

$$U = \xi_L^\dagger \xi_M \xi_R, \tag{16}$$

where ξ_L, ξ_R, ξ_M are transforming as

$$\xi_L \to h_L \xi_L g_L^\dagger, \quad \xi_R \to h_R \xi_L g_R^\dagger, \quad \xi_M \to h_L \xi_M h_R^\dagger, \quad h_L \in H_L, \quad h_R \in H_R.$$

Corresponding to this symmetry, BKY introduced gauge fields L_μ and R_μ:

$$D_\mu \xi_L = \partial \xi_L - iL_\mu \xi_L + i\xi_L \mathcal{L}_\mu, \quad D_\mu \xi_R = \partial \xi_R - iR_\mu \xi_R + i\xi_R \mathcal{R}_\mu, \quad D_\mu \xi_M = \partial \xi_M - iL_\mu \xi_M + i\xi_M R_\mu.$$

The BKY lagrangian is written as:

$$\begin{aligned}
\mathcal{L}_{\rm BKY} &= a f_\pi^2 {\rm tr}\left(\hat\beta_{\mu\parallel}\xi_M^\dagger \hat\beta_\parallel^\mu \xi_M^\dagger\right) + b f_\pi^2 {\rm tr}\left(\left(\hat\beta_{\mu\perp}\xi_M^\dagger + \frac{1}{2}\hat\beta_{\mu M}\right)^2\right) + \frac{c}{4} f_\pi^2 {\rm tr}\left(\hat\beta_{\mu M}\hat\beta_M^\mu\right) \\
&+ d f_\pi^2 {\rm tr}\left(\hat\beta_{\mu\perp}\xi_M^\dagger \hat\beta_\perp^\mu \xi_M^\dagger\right) - \frac{1}{4g^2}\left[{\rm tr}\left(L_{\mu\nu}L^{\mu\nu}\right) + {\rm tr}\left(R_{\mu\nu}R^{\mu\nu}\right)\right],
\end{aligned} \tag{17}$$

where $\hat\beta_{\mu\parallel}$,$\hat\beta_{\mu\perp}$ and $\hat\beta_{\mu M}$ are given by

$$\hat\beta_{\mu\parallel} \equiv \frac{1}{2}(\hat\beta_{\mu L}\xi_M + \xi_M \hat\beta_{\mu R}), \qquad \hat\beta_{\mu\perp} \equiv \frac{1}{2}(\hat\beta_{\mu L}\xi_M - \xi_M \hat\beta_{\mu R} - \hat\beta_{\mu M}\xi_M),$$

and

$$\hat{\beta}_{\mu L} \equiv \frac{1}{i} D_\mu \xi_L \cdot \xi_L^\dagger, \quad \hat{\beta}_{\mu R} \equiv \frac{1}{i} D_\mu \xi_R \cdot \xi_R^\dagger, \quad \hat{\beta}_{\mu M} \equiv \frac{1}{i} D_\mu \xi_M \cdot \xi_M^\dagger.$$

The linear combination $L_\mu \pm R_\mu$ correspond to ρ and a_1 mesons respectively.

To show the equivalence of the two formulations, we first rewrite the lagrangian (15) so as to make the generalized hidden local symmetry manifest:

$$\begin{aligned}\mathcal{L}_{\mathrm{AST}} \;=\;& \frac{f_\pi^2}{4}\mathrm{tr}((D^\mu U)^\dagger (D_\mu U)) \\ & -\frac{1}{2}\mathrm{tr}(\nabla^\lambda \mathbf{A}_{\lambda\mu}\xi_M^\dagger \nabla_\nu \mathbf{A}^{\nu\mu}\xi_M^\dagger) + \frac{M_A^2}{4}\mathrm{tr}(\mathbf{A}_{\mu\nu}\xi_M^\dagger \mathbf{A}^{\mu\nu}\xi_M^\dagger) - \frac{F_A}{\sqrt{2}}\mathrm{tr}(\mathbf{A}^{\mu\nu}\xi_M^\dagger \hat{\mathcal{A}}_{\mu\nu}\xi_M^\dagger), \\ & -\frac{1}{2}\mathrm{tr}(\nabla^\lambda \mathbf{V}_{\lambda\mu}\xi_M^\dagger \nabla_\nu \mathbf{V}^{\nu\mu}\xi_M^\dagger) + \frac{M_V^2}{4}\mathrm{tr}(\mathbf{V}_{\mu\nu}\xi_M^\dagger \mathbf{V}^{\mu\nu}\xi_M^\dagger) + \frac{F_V}{\sqrt{2}}\mathrm{tr}(\mathbf{V}^{\mu\nu}\xi_M^\dagger \hat{\mathcal{V}}_{\mu\nu}\xi_M^\dagger) \\ & +\frac{G_V}{2\sqrt{2}} i\,\mathrm{tr}(\mathbf{V}^{\mu\nu}\xi_M^\dagger [u_\mu \xi_M^\dagger, u_\nu \xi_M^\dagger]),\end{aligned} \tag{18}$$

where the covariant derivative ∇_μ and the chiral covariant one form u_μ are redefined as:

$$\nabla^\mu \mathbf{V}_{\mu\nu} = \partial^\mu \mathbf{V}_{\mu\nu} + \Gamma_L^\mu \mathbf{V}_{\mu\nu} - \mathbf{V}_{\mu\nu}\Gamma_R^\mu, \qquad \nabla^\mu \mathbf{A}_{\mu\nu} = \partial^\mu \mathbf{A}_{\mu\nu} + \Gamma_L^\mu \mathbf{A}_{\mu\nu} - \mathbf{A}_{\mu\nu}\Gamma_R^\mu,$$

and

$$u_\mu = i\xi_L (D_\mu U)\xi_R^\dagger$$

with

$$\begin{aligned}\Gamma_L^\mu &= -\frac{1}{2}\left[\partial^\mu \xi_L \cdot \xi_L^\dagger + \partial^\mu(\xi_M \xi_R)\cdot \xi_R^\dagger \xi_M^\dagger + i\xi_L \mathcal{L}^\mu \xi_L^\dagger + i\xi_M \xi_R \mathcal{R}^\mu \xi_R^\dagger \xi_M^\dagger\right], \\ \Gamma_R^\mu &= -\frac{1}{2}\left[\partial^\mu \xi_R \cdot \xi_R^\dagger + \partial^\mu(\xi_M^\dagger \xi_L)\cdot \xi_L^\dagger \xi_M + i\xi_R \mathcal{R}^\mu \xi_R^\dagger + i\xi_M^\dagger \xi_L \mathcal{L}^\mu \xi_L^\dagger \xi_M\right].\end{aligned}$$

The external fields $\hat{\mathcal{V}}_{\mu\nu}$ and $\hat{\mathcal{A}}_{\mu\nu}$ are redefined as

$$\hat{\mathcal{V}}_{\mu\nu} = \frac{1}{2}(\xi_L \mathcal{L}_{\mu\nu}\xi_L^\dagger \xi_M + \xi_M \xi_R \mathcal{R}_{\mu\nu}\xi_R^\dagger), \quad \hat{\mathcal{A}}_{\mu\nu} = \frac{1}{2}(-\xi_L \mathcal{L}_{\mu\nu}\xi_L^\dagger \xi_M + \xi_M \xi_R \mathcal{R}_{\mu\nu}\xi_R^\dagger).$$

The "matter" fields $\mathbf{V}_{\mu\nu}$ and $\mathbf{A}_{\mu\nu}$ transform as

$$\mathbf{V}_{\mu\nu} \to h_L \mathbf{V}_{\mu\nu} h_R^\dagger, \qquad \mathbf{A}_{\mu\nu} \to h_L \mathbf{A}_{\mu\nu} h_R^\dagger,$$

in the lagrangian (18). It is easy to see that (18) actually reproduces its original form (15) in the unitary gauge of the generalized hidden local symmetry, i.e., $\xi_M = 1$ and $\xi_L^\dagger = \xi_R$. We also obtain following relations:

$$\begin{aligned}\Gamma_L^\mu &= -\frac{i}{2}(\hat{\beta}_L^\mu + \xi_M \hat{\beta}_R^\mu \xi_M^\dagger + \hat{\beta}_M^\mu + 2L_\mu), \\ \Gamma_R^\mu &= -\frac{i}{2}(\hat{\beta}_R^\mu + \xi_M^\dagger \hat{\beta}_L^\mu \xi_M - \xi_M^\dagger \hat{\beta}_M^\mu \xi_M + 2R_\mu), \\ u_\mu &= 2\hat{\beta}_{\mu\perp}.\end{aligned}$$

The equivalence of the lagrangian (18) and the BKY formalism (17) can be shown in a similar manner to the case of the ρ meson. We introduce spin 1 fields L_μ and R_μ as auxiliary fields:

$$\begin{aligned}\mathcal{L}'_{\rm AST} &= \mathcal{L}_{\rm AST} \\ &+\frac{\kappa^2}{4}{\rm tr}\left[\left(L_\mu + i\partial_\mu\xi_L\cdot\xi_L^\dagger - \xi_L\mathcal{L}_\mu\xi_L^\dagger - \frac{1}{\kappa}\nabla^\lambda\mathbf{V}_{\lambda\mu}\cdot\xi_M^\dagger + \frac{1}{\kappa}\nabla^\lambda\mathbf{A}_{\lambda\mu}\cdot\xi_M^\dagger\right)^2\right] \\ &+\frac{\kappa^2}{4}{\rm tr}\left[\left(R_\mu + i\partial_\mu\xi_R\cdot\xi_R^\dagger - \xi_R\mathcal{R}_\mu\xi_R^\dagger - \frac{1}{\kappa}\xi_M^\dagger\cdot\nabla^\lambda\mathbf{V}_{\lambda\mu} - \frac{1}{\kappa}\xi_M^\dagger\cdot\nabla^\lambda\mathbf{A}_{\lambda\mu}\right)^2\right],\end{aligned} \tag{19}$$

with κ being parameters corresponding the arbitrariness of the definition of the effective spin 1 meson fields. By performing partial integration and using the identities

$$\begin{aligned}D_\mu\hat\beta_{\nu L} - D_\nu\hat\beta_{\mu L} &= -L_{\mu\nu} + \xi_L\mathcal{L}_{\mu\nu}\xi_L^\dagger + i[\hat\beta_{\mu L},\hat\beta_{\nu L}], \\ D_\mu\hat\beta_{\nu R} - D_\nu\hat\beta_{\mu R} &= -R_{\mu\nu} + \xi_R\mathcal{R}_{\mu\nu}\xi_R^\dagger + i[\hat\beta_{\mu R},\hat\beta_{\nu R}], \\ D_\mu\hat\beta_{\nu M} - D_\nu\hat\beta_{\mu M} &= -L_{\mu\nu} + \xi_M R_{\mu\nu}\xi_R^\dagger + i[\hat\beta_{\mu M},\hat\beta_{\nu M}],\end{aligned}$$

we find (19) can be rewritten as

$$\begin{aligned}\mathcal{L}'_{\rm AST} &= \frac{\kappa^2}{4}{\rm tr}(\hat\beta_{\mu L}\hat\beta_L^\mu) + \frac{\kappa^2}{4}{\rm tr}(\hat\beta_{\mu R}\hat\beta_R^\mu) + f_\pi^2{\rm tr}(\hat\beta_{\mu\perp}\xi_M^\dagger\hat\beta_\perp^\mu\xi_M^\dagger) \\ &+\frac{M_V^2}{4}{\rm tr}\left[(\mathbf{V}_{\mu\nu}\xi_M^\dagger + X_{V\mu\nu}\xi_M^\dagger)^2\right] + \frac{M_A^2}{4}{\rm tr}\left[(\mathbf{A}_{\mu\nu}\xi_M^\dagger + X_{A\mu\nu}\xi_M^\dagger)^2\right] \\ &-\frac{M_V^2}{4}{\rm tr}\left[X_{V\mu\nu}\xi_M^\dagger X_V^{\mu\nu}\xi_M^\dagger\right] - \frac{M_A^2}{4}{\rm tr}\left[X_{A\mu\nu}\xi_M^\dagger X_A^{\mu\nu}\xi_M^\dagger\right],\end{aligned} \tag{20}$$

where X_V and X_A are defined by

$$\begin{aligned}X_V^{\mu\nu} &\equiv \sqrt{2}F_V\hat{\mathcal{V}}^{\mu\nu} + 2\sqrt{2}G_V i[\hat\beta_\perp^\mu\xi_M^\dagger,\hat\beta_\perp^\nu\xi_M^\dagger]\xi_M - \frac{\kappa}{2}\left(2\hat{\mathcal{V}}^{\mu\nu} - L^{\mu\nu}\xi_M - \xi_M R^{\mu\nu}\right) \\ &+\frac{\kappa}{2}\left(i[\hat\beta_L^\mu,\xi_M\hat\beta_R^\nu\xi_M^\dagger]\xi_M + \frac{i}{2}[\hat\beta_M^\mu,\hat\beta_L^\nu - \xi_M\hat\beta_R^\nu\xi_M^\dagger]\xi_M - (\mu\leftrightarrow\nu)\right), \\ X_A^{\mu\nu} &\equiv -\sqrt{2}F_A\hat{\mathcal{A}}^{\mu\nu} + \frac{\kappa}{2}\left(2\hat{\mathcal{A}}^{\mu\nu} + L^{\mu\nu}\xi_M - \xi_M R^{\mu\nu}\right) \\ &+\frac{\kappa}{2}\left(\frac{i}{2}[\hat\beta_M^\mu,\hat\beta_L^\nu + \xi_M\hat\beta_R^\nu\xi_M^\dagger]\xi_M - (\mu\leftrightarrow\nu)\right),\end{aligned}$$

It is easy to integrate out the anti-symmetric tensor fields $\mathbf{V}_{\mu\nu}$ and $\mathbf{A}_{\mu\nu}$ from (20). We obtain

$$\begin{aligned}\mathcal{L}'_{\rm AST} &= \frac{\kappa^2}{4}{\rm tr}(\hat\beta_{\mu L}\hat\beta_L^\mu) + \frac{\kappa^2}{4}{\rm tr}(\hat\beta_{\mu R}\hat\beta_R^\mu) + f_\pi^2{\rm tr}(\hat\beta_{\mu\perp}\xi_M^\dagger\hat\beta_\perp^\mu\xi_M^\dagger) \\ &-\frac{M_V^2}{4}{\rm tr}\left[X_{V\mu\nu}\xi_M^\dagger X_V^{\mu\nu}\xi_M^\dagger\right] - \frac{M_A^2}{4}{\rm tr}\left[X_{A\mu\nu}\xi_M^\dagger X_A^{\mu\nu}\xi_M^\dagger\right].\end{aligned} \tag{21}$$

The expression (21) is invariant under generalized hidden local symmetry and includes the spin 1 fields as gauge fields. The lagrangian (21) includes left-right gauge field kinetic mixing term $\mathrm{tr}(L_{\mu\nu}\xi_M R^{\mu\nu}\xi_M^\dagger)$, which is not included in the BKY formalism, however.

We have three ambiguities of the definition of the effective fields L_μ and R_μ corresponding to the three chiral covariant one forms $\hat{\beta}_{\mu L}$, $\hat{\beta}_{\mu R}$ and $\hat{\beta}_{\mu M}$. One ambiguity is already taken into account by the parameter κ. The following redefinition is convenient to parametrize the rest of the ambiguities:

$$
\begin{aligned}
L_\mu &\to L_\mu + \lambda\mu\hat{\beta}_{\mu L} - (\lambda-\mu)\xi_M\hat{\beta}_{\mu R}\xi_M^\dagger + (1-\lambda)\mu\hat{\beta}_{\mu M}, \\
R_\mu &\to R_\mu + \lambda\mu\hat{\beta}_{\mu R} - (\lambda-\mu)\xi_M^\dagger\hat{\beta}_{\mu L}\xi_M - (1-\lambda)\mu\xi_M^\dagger\hat{\beta}_{\mu M}\xi_M,
\end{aligned}
$$

where λ and μ are arbitrary parameters. We can eliminate the above mentioned term $\mathrm{tr}(L_{\mu\nu}\xi_M R^{\mu\nu}\xi_M^\dagger)$ by using the parameter λ. The parameter μ represents the rest of the ambiguity.

A plausible choice of the parameters κ,λ and μ is to determine them so as to eliminate the gauge kinetic mixing terms, e.g, $\mathrm{tr}(L_{\mu\nu}\xi_L\mathcal{L}^{\mu\nu}\xi_L^\dagger)$ in the effective lagrangian. This particular choice of effective field definition leads to

$$
\kappa = -\frac{M_A - M_V}{2\sqrt{2}}\left(\frac{F_V}{M_V} + \frac{F_A}{M_A}\right), \quad \lambda = -\frac{M_A + M_V}{M_A - M_V}, \quad \mu = -\frac{F_V M_A - F_A M_V}{F_V M_A + F_A M_V}. \tag{22}
$$

The lagrangian (21) then becomes

$$
\begin{aligned}
\mathcal{L}'_{\mathrm{AST}} &= -\frac{F_V^2}{4M_V^2}\left[\mathrm{tr}(L_{\mu\nu}L^{\mu\nu}) + \mathrm{tr}(R_{\mu\nu}R^{\mu\nu})\right] \\
&\quad + F_V^2\,\mathrm{tr}\left(\hat{\beta}_{\mu\|}\xi_M^\dagger\hat{\beta}_\|^\mu\xi_M^\dagger\right) + F_V F_A\frac{M_A}{M_V}\mathrm{tr}\left(\left(\hat{\beta}_{\mu\perp}\xi_M^\dagger + \frac{1}{2}\hat{\beta}_{\mu M}\right)^2\right) \\
&\quad + \frac{1}{4}\left[F_V^2\frac{M_A^2}{M_V^2} - F_V F_A\frac{M_A}{M_V}\right]\mathrm{tr}\left(\hat{\beta}_{\mu M}\hat{\beta}_M^\mu\right) + \left[f_\pi^2 + F_A^2 - F_V F_A\frac{M_A}{M_V}\right]\mathrm{tr}\left(\hat{\beta}_{\mu\perp}\xi_M^\dagger\hat{\beta}_\perp^\mu\xi_M^\dagger\right) \\
&\quad + \cdots,
\end{aligned} \tag{23}
$$

where $\cdots$ stands for higher derivative terms in the hidden local symmetry formulation. We thus obtain explicit relations between the anti-symmetric tensor method and the

generalized hidden local symmetry formalism:

$$\begin{aligned} g &= \frac{M_V}{F_V}, & a &= \frac{F_V^2}{f_\pi^2}, \\ b &= \frac{F_V F_A}{f_\pi^2}\frac{M_A}{M_V}, & c &= \frac{F_V^2 M_A^2}{f_\pi^2 M_V^2} - \frac{F_V F_A M_A}{f_\pi^2 M_V}, \\ d &= 1 + \frac{F_A^2}{f_\pi^2} - \frac{F_V F_A}{f_\pi^2}\frac{M_A}{M_V}. \end{aligned} \tag{24}$$

Plugging the phenomenological value $F_V^2 \simeq 2 f_\pi^2$ and the Weinberg sum rules $f_\pi^2 = F_V^2 - F_A^2$ and $F_V^2 M_V^2 = F_A^2 M_A^2$ in (24) , we obtain the coefficients $a = b = c = 2$, $d = 0$, in agreement with the values quoted in Ref.[9].

In this paper, we have shown how the auxiliary field method works to clarify the relation of the anti-symmetric tensor and the hidden local symmetry formalisms of the ρ and the a_1 mesons. The ambiguity of the definition of the effective fields in the hidden local symmetry formalism can be resolved by using the extra conditions, e.g., the disappearance of the kinetic mixing terms in the effective lagrangian. The anti-symmetric tensor field method is equivalent to the hidden local symmetry lagrangian plus on-shell KSRF I violating $\mathcal{O}(E^4)$ term. For analysis of non QCD-like technicolor models, this term might become important.

The author thanks M. S. Chanowitz, Y. Okada, M. Suzuki and K. Yamawaki for enlightening discussions. He is also grateful to B. Bullock for careful reading of the manuscript.

Note added:

After the manuscript has been completed, I realized similar work of J. Bijnens and E. Pallante[19].

References

[1] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158** (1984) 142; Nucl. Phys. **B250** (1985) 465.

[2] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321** (1989) 311; J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. **D39** (1989) 1947.

[3] B. Holdom, Phys. Lett. **B258** (1991) 156.

[4] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65** (1990) 964; B. Holdom and J. Terning, Phys. Lett. **B247** (1990) 88.

[5] K. Hagiwara, R.D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. **B262** (1987) 233.

[6] B. Holdom, Phys. Lett. **B150** (1985) 301; K. Yamawaki, M. Bando and K. Matumoto, Phys. Rev. Lett. **56** (1986) 1335; T. Akiba and T. Yanagida, Phys. Lett. **B169** (1986) 432; T. Appelquist, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. Lett. **57** (1986) 957.

[7] E.H. Simmons, Nucl. Phys. **B312** (1989) 253.

[8] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54** (1985) 1215.

[9] For a review, M. Bando, T. Kugo and K. Yamawaki, Phys. Reports **164** (1988) 218.

[10] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16** (1966) 255; Riazuddin and Fayyazuddin, Phys. Rev. **147** (1966) 1071.

[11] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. **B259** (1985) 493.

[12] M. Bando, T. Kugo and K. Yamawaki, Prog. Theor. Phys. **73** (1985) 1541.

[13] M. Harada and K. Yamawaki, Phys. Lett. **B297** (1992) 151; M. Harada, T. Kugo and K. Yamawaki, Phys. Rev. Lett. **71** (1993) 1299; Prog. Theor. Phys. **91** (1994) 801.

[14] H. Georgi, Phys. Rev. Lett. **63** (1989) 1917; Nucl. Phys. **B331** (1990) 311.

[15] M. Tanabashi, Phys. Lett. **B316** (1993) 534.

[16] R. Casalbuoni, S. de Curtis, D. Dominici and R. Gatto, Phys. Lett. **B155**, (1985) 95; Nucl. Phys. **B282**, (1987) 235.

[17] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. **B223** (1989) 425.

[18] D. Kalafatis, Phys. Lett. **B313**, (1993) 115.

[19] J. Bijnens and E. Pallante, NORDITA preprint NORDITA-95/63 N,P, hep-ph/9510338.